Chapter 3: Motion in a Plane

EXERCISES [PAGES 44 - 46]

Exercises | Q 1. (i) | Page 44

Choose the correct option.
An object thrown from a moving bus is on example of _____.

1. Uniform circular motion
2. Rectilinear motion
3. **Projectile motion**
4. Motion in one dimension

SOLUTION

An object thrown from a moving bus is on an example of **Projectile motion.**

Exercises | Q 1. (ii) | Page 44

Choose the correct option.
For a particle having a uniform circular motion, which of the following is constant

1. **Speed**
2. Acceleration
3. Velocity
4. Displacement

SOLUTION

Speed

Exercises | Q 1. (iii) | Page 44

Choose the correct option.
The bob of a conical pendulum undergoes

1. Rectilinear motion in horizontal plane
2. **Uniform motion in a horizontal circle**
3. Uniform motion in a vertical circle
4. Rectilinear motion in vertical circle

SOLUTION

Uniform motion in a horizontal circle

Exercises | Q 1. (iv) | Page 44

Choose the correct option.

For uniform acceleration in rectilinear motion which of the following is not correct?

1. Velocity-time graph is linear
2. Acceleration is the slope of velocity-time graph
3. The area under the velocity-time graph equals displacement
4. **Velocity-time graph is nonlinear**

SOLUTION

Velocity-time graph is nonlinear

Exercises | Q 1. (v) | Page 44

Choose the correct option.

If three particles A, B and C are having velocities $\vec{V}_A + \vec{V}_B$ and $\vec{V}_C$ which of the following formula gives the relative velocity of A with respect to B?

$\vec{V}_A + \vec{V}_B$

$\vec{V}_A - \vec{V}_C + \vec{V}_B$

$\vec{V}_A - \vec{V}_B$

$\vec{V}_C + \vec{V}_A$

SOLUTION

$$\vec{V}_A - \vec{V}_B$$

Exercises | Q 2. (i) | Page 45

Answer the following question.

Separate the following in groups of scalar and vectors:
velocity, speed, displacement, work done, force, power, energy, acceleration, electric charge, angular velocity

SOLUTION

Scalars	Vectors
Speed, work done, power, energy, electric charge	Velocity, displacement, force, acceleration, angular velocity (pseudo vector)

Exercises | Q 2. (ii) | Page 45

Answer the following question.

Define average velocity and instantaneous velocity. When are they same?

SOLUTION

Average velocity:

1. Average velocity $\left(\vec{v}_{av}\right)$ of an object is the displacement $\left(\Delta\vec{X}\right)$ of the object during the time interval (Δt) over which average velocity is being calculated, divided by that time interval.
2. Average velocity = $\left(\frac{\text{Displacement}}{\text{Time interval}}\right)$
$$\vec{v}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{\Delta\vec{x}}{\Delta t}$$
3. Average velocity is a vector quantity.
4. Its SI unit is m/s and dimensions are $[M^0L^1T^{-1}]$
5. For example, if the positions of an object are x = +4 m and x = +6 m at times t = 0 and t = 1 minute respectively, the magnitude of its average velocity during that time is $\mathbf{v_{av}}$ = (6 - 4)/(1 - 0) = 2 m per minute and its direction will be along the positive X-axis.
$\therefore\ \vec{v}_{av}$ = 2 i m/min
where, i = unit vector along X-axis.

Instantaneous velocity:

1. The instantaneous velocity $\left(\vec{v}\right)$ is the limiting value of the average velocity of the object over a small time interval (Δt) nd t when the value of time interval goes to zero.
2. It is the velocity of an object at a given instant of time.
3. $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$ where
$\frac{d\vec{x}}{dt}$ = derivative of $\vec{x}$ with respect to t.

In the case of uniform rectilinear motion, i.e., when an object is moving with constant velocity along a straight line, the average and instantaneous velocity remain the same.

Exercises | Q 2. (iii) | Page 45

Answer the following question.
Define free fall.

SOLUTION

The motion of any object under the influence of gravity alone is called as free fall.

Exercises | Q 2. (iv) | Page 45

Answer the following question.

If the motion of an object is described by x = f(t), write formulae for instantaneous velocity and acceleration.

SOLUTION

1. Instantaneous velocity of an object is given as,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

2. Motion of the object is given as, x= f(t)
3. The derivative f '(t) represents the rate of change of the position f (t) at time t, which is the instantaneous velocity of the object.

$$\therefore \vec{v} = \frac{d\vec{x}}{dt} = f'(t)$$

4. Acceleration is defined as the rate of change of velocity with respect to time.
5. The second derivative of the position function f ''(t) represents the rate of change of velocity i.e., acceleration.

$$\therefore \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{d^2x}{dt^2} = f''(t)$$

Exercises | Q 2. (v) | Page 45

Answer the following question.

Derive equations of motion for a particle moving in a plane and show that the motion can be resolved in two independent motions in mutually perpendicular directions.

SOLUTION

1. Consider an object moving in an x-y plane. Let the initial velocity of the object be $\vec{u}$ at t = 0 and its velocity at time t be $\vec{v}$
2. As the acceleration is constant, the average acceleration and the instantaneous acceleration will be equal.

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \left(\frac{v_{2x} - v_{1x}}{t_2 - t_1}\right)\hat{i} + \left(\frac{v_{2y} - v_{1y}}{t_2 - t_1}\right)$$

$$\therefore \vec{a} = \frac{\left(\vec{v} - \vec{u}\right)}{(t - 0)}$$

$$\therefore \vec{v} = \vec{u} + \vec{a}t \quad ....(1)$$

This is the first equation of motion in vector form.

3. Let the displacement of the object from time t = 0 to t be $\vec{s}$

For constant acceleration, $\vec{v}_{av} = \frac{\vec{v} + \vec{u}}{2}$

$$\vec{s} = \left(\vec{v}_{av}\right)t = \left(\frac{\vec{v} + \vec{u}}{2}\right)t = \left(\frac{\vec{u} + \vec{u} + \vec{a}t}{2}\right)t$$

$$\therefore \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad ...(2)$$

This is the second equation of motion in vector form.

4. Equations (1) and (2) can be resolved into their x and y components so as to get corresponding scalar equations as follows.

$$v_x = u_x + a_x t \quad ....(3)$$
$$v_y = u_y + a_y t \quad ....(4)$$
$$s_x = u_x t + \frac{1}{2} a_x t^2 \quad ...(5)$$
$$s_y = u_y t + \frac{1}{2} a_y t^2 \quad ....(6)$$

5. It can be seen that equations (3) and (5) involve only the x components of displacement, velocity and acceleration while equations (4) and (6) involve only the y components of these quantities.

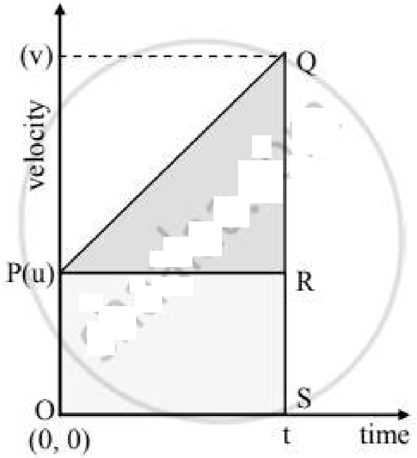
6. Thus, the motion along the x-direction of the object is completely controlled by the x components of velocity and acceleration while that along the y-direction is completely controlled by the y components of these quantities.

7. This shows that the two sets of equations are independent of each other and can be solved independently.

Exercises | Q 2. (vi) | Page 45

Answer the following question.
Derive equations of motion graphically for a particle having uniform acceleration, moving along a straight line.

SOLUTION



1. Consider an object starting from position x = 0 at time t = 0. Let the velocity at time (t = 0) and t be u and v respectively.

2. The slope of line PQ gives the acceleration. Thus

$$\therefore a = \frac{v - u}{t - 0} = \frac{v - u}{t}$$

$\therefore v = u + at$ ....(1)

This is the first equation of motion.

3. The area under the curve in velocity-time graph gives the displacement of the object.

$\therefore$ s = area of the quadrilateral OPQS

= area of rectangle OPRS + area of triangle PQR

$$= ut + \frac{1}{2}(v - u)t$$

But, from equation (1)

$at = v - u$

$$\therefore s = ut + \frac{1}{2}at^2$$

This is the second equation of motion.

4. The velocity is increasing linearly with time as acceleration is constant. The displacement is given as,

$$s = v_{av}t = \left(\frac{v + u}{2}\right)t$$

$$= \frac{(v + u)(v - u)}{2(v - u)}$$

$$= \frac{(v + u)(v - u)}{2a}$$

$\therefore s = (v^2 - u^2)//(2a)$

$\therefore v^2 - u^2 = 2as$

This is the third equation of motion.

Exercises | Q 2. (vii) | Page 45

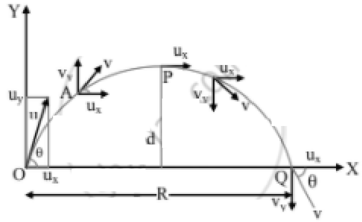
Answer the following question.

Derive the formula for the range and maximum height achieved by a projectile thrown from the origin with initial velocity u at an angle θ to the horizontal.

SOLUTION

1. Consider a body projected with velocity $\vec{u}$, at an angle θ of projection from point O in the coordinate system of the XY- plane, as shown in the figure.
2. The initial velocity $\vec{u}$ can be resolved into two rectangular components:

2. The initial velocity $\vec{u}$ can be resolved into two rectangular components:



Trajectory of a projectile

u_x = u cos θ (Horizontal component)

u_y = u sin θ (Vertical component)

3. The horizontal component remains constant throughout the motion due to the absence of any force acting in that direction, while the vertical component changes according to

$v_y = u_y + a_y t$

with $a_y = -g$ and $u_y = u \sin \theta$

4. Thus, the components of velocity of the projectile at time t are given by,

$v_x = u_y + a_y t$

$v_y = uy - gt = u \sin \theta - gt$

5. Similarly, the components of displacements of the projectile in the horizontal and vertical directions at time t are given by,

$s_x = (u \cos \theta)t$

$s_y = (u \sin \theta)t - \frac{1}{2}gt^2$ ....(1)

6. At the highest point, the time of ascent of the projectile is given as, $t_A = \frac{u \sin\theta}{g}$ ....(2)

7. The total time in air i.e., time of flight is given as, $T = 2t_A = \frac{2u \sin\theta}{g}$ .....(3)

8. The total horizontal distance travelled by the particle in this time T is given as,

$R = u_x \cdot T$

$R = u \cos \theta * (2t_A)$

$\therefore R = u \cos \theta \cdot \frac{2u \sin\theta}{g}$ ....[From(3)]

$\therefore R = \frac{u^2(2 \sin\theta \cdot \cos\theta)}{g}$

$\therefore R = \frac{u^2 \sin 2\theta}{g}$ ....[$\because \sin 2\theta = 2 \sin \theta * \cos \theta$]

This is a required expression for the horizontal range of the projectile.

Expression for a maximum height of a projectile:

1. The maximum height H reached by the projectile is the distance travelled along the vertical (y) direction in time t_A.
2. Substituting s_y = H and t = t_a in equation (1), we have,

$$H = (u \sin\theta)t_A - \frac{1}{2}gt_A^2$$

$$\therefore H = u\sin\theta\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2 \quad \text{....[From (2)]}$$

$$\therefore H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

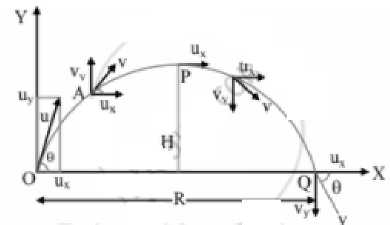
This equation represents the maximum height of projectile.

Exercises | Q 2. (viii) | Page 45

Answer the following question.
Show that the path of a projectile is a parabola.

SOLUTION

1. Consider a body projected with velocity initial velocity $\vec{u}$, at an angle θ of projection from point O in the co-ordinate system of the XY-plane, as shown in the figure.



Trajectory of a projectile

2. The initial velocity $\vec{u}$ can be resolved into two rectangular components:

   $u_x = u \cos\theta$ (Horizontal component)

   $u_y = u \sin\theta$ (Vertical component)

3. The horizontal component remains constant throughout the motion due to the absence of any force acting in that direction, while the vertical component changes according to,

   $v_y = u_y + a_y t$

   with $a_y = -g$ and $u_y = u \sin\theta$

4. Thus, the components of velocity of the projectile at time t are given by,

   $v_x = u_x = u \cos\theta$

   $v_y = u_y - gt = u \sin\theta - gt$

5. Similarly, the components of displacements of the projectile in the horizontal and vertical directions at time t are given by,

   $s_x = (u \cos \theta)t$ ...(1)

   $s_y = (u \sin \theta)t - \frac{1}{2}gt^2$ ....(2)

6. As the projectile starts from x = 0, we can use

   $s_x = x$ and $s_y = y$

   Substituting $s_x = x$ in equation (1),

   $x = (u \cos \theta)t$

   $\therefore t = \frac{x}{u \cos \theta}$ ...(3)

   Substituting, $s_y = y$ in equation (2),

   $y = (u \sin \theta)t - \frac{1}{2}gt^2$ ....(4)

   Substituting equation (3) in equation (4), we have,

   $$y = u \sin \theta \left(\frac{x}{u \sin \theta}\right) - \frac{1}{2}\left(\frac{x}{u \cos \theta}\right)^2 g$$

   $\therefore y = x (\tan \theta) - \left(\frac{g}{2u^2 \cos^2 \theta}\right)x^2$ ....(5)

   Equation (5) represents the path of the projectile.

7. If we put $\tan \theta = A$ and $g/2u^2 \cos^2\theta = B$ then equation (5) can be written as $y = Ax - Bx^2$ where A and B are constants. This is the equation of a parabola. Hence, the path of the projectile is a parabola.

Exercises | Q 2. (ix) | Page 45

Answer the following question.

What is a conical pendulum?

SOLUTION

A simple pendulum, which is given such a motion that the bob describes a horizontal circle and the string making a constant angle with the vertical describes a cone, is called a conical pendulum.

Exercises | Q 2. (x) | Page 45

Answer the following question.

Define angular velocity.

SOLUTION

Angular velocity of a particle is the rate of change of angular displacement.

Exercises | Q 2. (xi) | Page 45

Answer the following question.

Show that the centripetal force on a particle undergoing uniform circular motion is $-mr\omega^2$.

1. Suppose a particle is performing U.C.M in anticlockwise direction. The co-ordinate axes are chosen as shown in the figure.

   Let, A = initial position of the particle which lies on positive X-axis

   P = instantaneous position after time t

   θ = angle made by radius vector

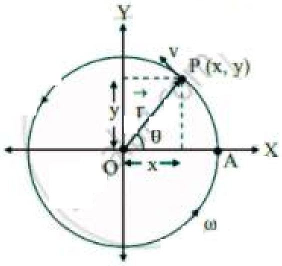
   ω = constant angular speed

   $\vec{r}$ = instantaneous position vector at time t

2. From the figure,

   $\vec{r} = \hat{i}x + \hat{j}y$

   where, $\hat{i}$ and $\hat{j}$ are unit vectors along X-axis and Y-axis respectively.



3. Also, x = r cos θ and y = r sin θ

   $$\therefore \vec{r} = \left[r\hat{i}\cos\theta + r\hat{j}\sin\theta\right]$$

   But θ = ωt

   $$\therefore \vec{r} = \left[r\hat{i}\cos\omega t + r\hat{j}\sin\omega t\right] \quad ....(1)$$

4. Velocity of the particle is given as rate of change of position vector.

   $$\therefore \vec{v} = \frac{d\vec{r}}{dt}\left[r\hat{i}\cos\omega t + r\hat{j}\sin\omega t\right]$$

   $$= r\left[\frac{d}{dt}\cos\omega t\right]\hat{i} + r\left[\frac{d}{dt}\sin\omega t\right]\hat{j}$$

   $$\therefore \vec{v} = -r\omega\hat{i}\sin\omega t + r\omega\hat{j}\cos\omega t$$

   $$\therefore \vec{v} = -r\omega\left(-\hat{i}\sin\omega t + \hat{j}\cos\omega t\right)$$

5. Further, instantaneous linear acceleration of the particle at instant t is given by,

   $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[r\omega\left(-\hat{i}\sin\omega t + \hat{j}\cos\omega t\right)\right]$$

$$= r\omega\left[\frac{d}{dt}\left(-\hat{i}\sin\omega t + \hat{j}\cos\omega t\right)\right]$$

$$= r\omega\left[\frac{d}{dt}(-\sin\omega t)\hat{i} + \frac{d}{dt}(\cos\omega t)\hat{j}\right]$$

$$= r\omega\left(-\omega\hat{i}\cos\omega t - \omega\hat{j}\sin\omega t\right)$$

$$= -r\omega^2\left(\hat{i}\cos\omega t + \hat{j}\sin\omega t\right)$$

$$\therefore \vec{a} = -\omega^2\left(r\hat{i}\cos\omega t + r\hat{j}\sin\omega t\right) \quad \text{....(2)}$$

6. From equation (1) and (2),

   $$\vec{a} = -\omega^2\vec{r} \quad \text{....(3)}$$

   The negative sign shows that the direction of acceleration is opposite to the direction of the position vector.

   Equation (3) is the centripetal acceleration.
7. The magnitude of centripetal acceleration is given by, a = $\omega^2 r$
8. The force providing this acceleration should also be in the same direction, hence centripetal.

   $$\therefore \vec{F} = m\vec{a} = -m\omega^2\vec{r}$$

   This is the expression for the centripetal force on a particle undergoing uniform circular motion.
9. Magnitude of F = $m\omega^2 r = \frac{mv^2}{r}$ = mωv

Exercises | Q 3. (i) | Page 45

Solve the following problem.

An aeroplane has a run of 500 m to take off from the runway. It starts from rest and moves with constant acceleration to cover the runway in 30 sec. What is the velocity of the aeroplane at the take-off?

SOLUTION

Given: Length of runway (s) = 500 m, t = 30 s

To find: Velocity (v)

Formulae: 1. s = ut + $\frac{1}{2}at^2$

2. v = u + at

Calculation: As the plane was initially at rest, u = 0 From formula (i),

500 = $0 + \frac{1}{2} \times a \times (30)^2$

$\therefore 500 = 450\ a$

$\therefore a = \frac{10}{9}\ m/s^2$

From formula (ii),

$v = 0 + \frac{10}{9} \times 30$

$\therefore v = \frac{100}{3} \mathrm{m/s} = \left(\frac{100}{3} \times \frac{18}{5}\right) \mathrm{km/hr}$

$\therefore v = 120$ km/hr

The velocity of the aeroplane at the take off is **120 km/hr**.

Exercises | Q 3. (ii) | Page 45

Solve the following problem.

A car moving along a straight road with a speed of 120 km/hr, is brought to rest by applying brakes. The car covers a distance of 100 m before it stops. Calculate
(i) the average retardation of the car
(ii) time taken by the car to come to rest.

SOLUTION

Given: $u = 120\ kmh^{-1} = 120 \times \frac{5}{18} = \frac{100}{3} \mathrm{ms}^{-1}$

$s = 100$ m, $v = 0$

To find: 1. Average retardation of the car (a)

2. Time taken by car (t)

Formulae: 1. $v^2 - u^2 = 2as$

2. $v = u + at$

Calculation: From formula (i),

$0 - \left(\frac{100}{3}\right)^2 = 2a \times 100$

$\therefore a = \frac{-10000}{9} \times \frac{1}{200} = -\frac{50}{9} \mathrm{ms}^{-2}$

$\therefore$ a = $\frac{-10000}{9} \times \frac{1}{200} = -\frac{50}{9}$ ms^{-2}

From formula (ii),

t = $\frac{\mathbf{v - u}}{\mathbf{a}} = \frac{0 - \frac{100}{3}}{-\frac{50}{9}}$ = 6 s

1. Average retardation of the car is $\frac{50}{9} \mathrm{ms}^{-2}$ (in magnitude).
2. Time taken by the car to come to rest is **6 s**.

Exercises | Q 3. (iii) | Page 45

Solve the following problem.
A car travels at a speed of 50 km/hr for 30 minutes, at 30 km/hr for next 15 minutes and then 70 km/hr for next 45 minutes. What is the average speed of the car?

SOLUTION

Given:

v_1 = 50 km/hr, t_1 = 30 minutes = 0.5 hr,

v_2 = 30 km/hr, t_2 = 15 minutes = 0.25 hr,

v_3 = 70 km/hr, t_3 = 45 minutes = 0.75 hr

To find: Average speed of car (v_{av})

Formula: $v_{av} = \frac{\text{total path length}}{\text{total time interval}}$

Calculation: Path length,

$x_1 = v_1 \times t_1 = 50 \times 0.5 = 25$ km

$x_2 = v_2 \times t_2 = 30 \times 0.25 = 7.5$ km

$x_3 = v_3 \times t_3 = 70 \times 0.75 = 52.5$ km

From formula,

$$v_{av} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3}$$

$$\therefore v_{av} = \frac{25 + 7.5 + 52.5}{0.5 + 0.25 + 0.75} = \frac{85}{1.5}$$

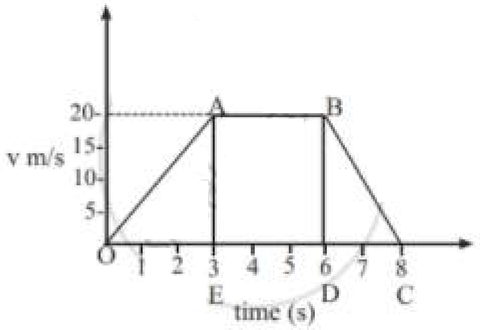
$\therefore v_{av}$ = 56.66 km/hr

Average speed of the car is **56.66 km/hr**.

Exercises | Q 3. (iv) | Page 45

Solve the following problem.

A velocity-time graph is shown in the adjoining figure.



Determine:

1. initial speed of the car
2. maximum speed attained by the car
3. part of the graph showing zero acceleration
4. part of the graph showing constant retardation
5. distance travelled by the car in first 6 sec.

SOLUTION

1. Initial speed is at origin i.e. **0 m/s**.
2. Maximum speed attained by car,
   v_{max} = speed from A to B = **20 m/s**.
3. The part of the graph which shows zero acceleration is between t = 3 s and t = 6 s i.e., AB. This is because, during **AB** there is no change in velocity.
4. The graph shows constant retardation from t = 6 s to t = 8 s i.e., **BC**.
5. Distance travelled by car in first 6 s

= Area of OABDO

= A(ΔOAE) + A(rect. ABDE

= $\frac{1}{2} \times 3 \times 20 + 3 \times 20$

= 30 + 60

∴ Distance travelled by car in first 6 s = **90 m.**

Exercises | Q 3. (v) | Page 45

Solve the following problem.
A man throws a ball to maximum horizontal distance of 80 m. Calculate the maximum height reached.

SOLUTION

Given: R_{max} = 80 m

To find: Maximum height reached (H_{max})

Formula: $R_{max} = 4H_{max}$

Calculation: From formula,

$$\therefore H_{max} = \frac{R_{max}}{4} = \frac{80}{4} = 20 \text{ m}$$

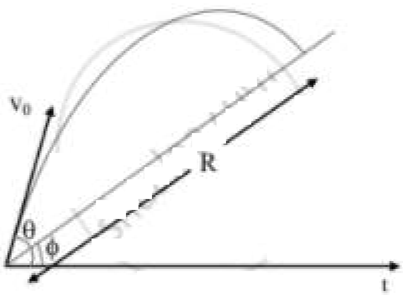
The maximum height reached by the ball is **20 m**.

Exercises | Q 3. (vi) | Page 45

Solve the following problem.

A particle is projected with speed v_0 at angle θ to the horizontal on an inclined surface making an angle Φ (Φ < θ) to the horizontal. Find the range of the projectile along the inclined surface.

SOLUTION



1. The equation of trajectory of projectile is given by,

$$y = (\tan\theta)x - \left[\frac{g}{2u^2\cos^2\theta}\right]x^2 \quad ...(1)$$

2. In this case to find R substitute,

$y = R \sin \Phi$ ....(2)

$x = R \cos \Phi$ ....(3)

3. From equations (1), (2) and (3), we have,

$$R \sin \Phi = \tan\theta (R \cos \Phi) - \left(\frac{g}{2v_0^2\cos^2\theta}\right) R^2\cos^2\phi \quad ....(\because u = v_0)$$

4. So, $\sin \Phi = \tan\theta \cos \Phi - \frac{g R\cos^2\phi}{2v_0^2\cos^2\theta}$

$$\therefore \frac{g R\cos^2\phi}{2v_0^2\cos^2\theta} = \tan\theta\cos\phi - \sin\phi$$

5. Hence,

$$R = \frac{2v_0^2}{g}\left[\frac{\cos^2\theta}{\cos^2\phi}\right][\tan\theta\cos\phi - \sin\phi]$$

$$= \frac{2v_0^2}{g}\frac{\cos\theta}{\cos^2\phi}\left[\cos\theta\frac{\sin\theta}{\cos\theta}\cos\phi - \cos\theta\sin\phi\right]$$

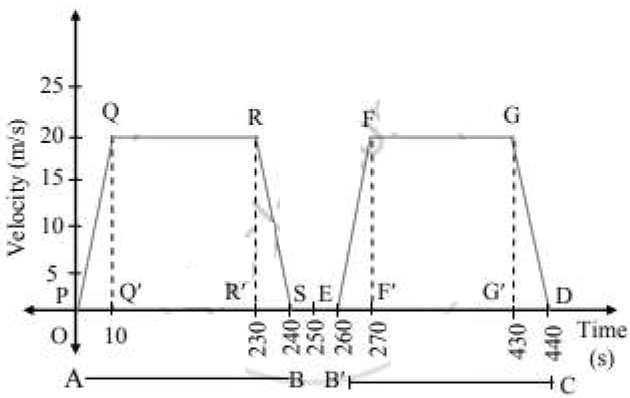
6. So, $R = \frac{2v_0^2}{g} = \frac{\cos\theta}{\cos^2\phi}[\sin\theta\cos\phi - \cos\theta\sin\phi]$

$$\therefore R = \frac{2v_0^2}{g} = \frac{\cos\theta}{\cos^2\phi}\sin(\theta-\phi) \quad .....[\because \sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi]$$

Exercises | Q 3. (vii) | Page 45

Solve the following problem.
A metro train runs from station A to B to C. It takes 4 minutes in travelling from station A to station B. The train halts at station B for 20 s. Then it starts at station B and reaches station C in next 3 minutes. At the start, the train accelerates for 10 sec to reach a constant speed of 72 km/hr. The train moving at the constant speed is brought to rest in 10 sec. At the next station.
(i) Plot the velocity- time graph for the train travelling from station A to B to C.
(ii) Calculate the distance between the stations A, B and C.

SOLUTION



The metro train travels from station A to station B in 4 minutes = 240 s.
The trains halts at station B for 20 s.

The train travels from station B' to station C in 3 minutes = 180 s.
∴ Total time taken by the metro train in travelling from station A to B to C
= 240 + 20 + 180 = 440 s.

At start, the train accelerates for 10 seconds to reach a constant speed of 72 km/hr = 20 m/s. The train moving is brought to rest in 10 s at next station.
The velocity-time graph for the train travelling from station A to B to C is as follows:

Distance travelled by the train from station A to station B

= Area of PQRS

$= A(\Delta\ PQQ') + A(\square\ Q'QRR') + A(SRR')$

$$= \left(\frac{1}{2} \times 10 \times 20\right) + (220 \times 20) + \left(\frac{1}{2} \times 10 \times 20\right)$$

= 100 + 4400 + 100

= 4600 m = **4.6 km**

∴ Distance travelled by the train from station B to station C

= Area of EFGD

= A(Δ EFF') + A(□ F'FGG') + A(Δ DGG')

$$= \left(\frac{1}{2} \times 10 \times 20\right) \times (160 \times 20) + \left(\frac{1}{2} \times 10 \times 20\right)$$

= 100 + 3200 + 100

= 3400 m = **3.4 km**

Exercises | Q 3. (viii) | Page 46

Solve the following problem.
A train is moving eastward at 10 m/sec. A waiter is walking eastward at 1.2m/sec; and a fly is charging toward the north across the waiter's tray at 2 m/s. What is the velocity of the fly relative to Earth?

SOLUTION

Given:

velocity of train w.r.t Earth, $\vec{v}_{TE} = 10\hat{i}$

velocity of waiter w.r.t train, $\vec{v}_{WT} = 1.2\hat{i}$

velocity of fly w.r.t waiter, $\vec{v}_{FW} = 2\hat{j}$

∴ Velocity of fly with respect to Earth

$$\vec{v}_{FE} = \vec{v}_{FT} - \vec{v}_{ET}$$

$$= \left(\vec{v}_{FW} - \vec{v}_{TW}\right) - \vec{v}_{ET}$$

$$= 2\hat{j} - \left(-1.2\hat{i}\right) - \left(-10\hat{i}\right)$$

$= 2\hat{j} + 11.2\hat{i}$ .....(considering north along +y axis)

Magnitude = $\sqrt{11.2^2 + 2^2}$

= 11.38 m/s ≈ **11.4 m/s**

Direction of velocity,

$$\theta = \tan^{-1}\left(\frac{2}{11.2}\right) \approx 10^{\circ} \text{ towards north of east.}$$

Exercises | Q 3. (ix) | Page 46

Solve the following problem.

A car moves in a circle at a constant speed of 50 m/s and completes one revolution in 40 s. Determine the magnitude of the acceleration of the car.

SOLUTION 1

Given: v = 50 m/s, t = 40 s, s = 2πr

To find: acceleration (a)

Formulae: i. $v = \frac{s}{t}$

ii. $a = \frac{v^2}{r}$

Calculation: From formula (i),

$$50 = \frac{2\pi r}{40}$$

$$\therefore r = \frac{50 \times 40}{2\pi}$$

$$\therefore r = \frac{1000}{\pi} \text{ cm}$$

From formula (ii),

$$a = \frac{v^2}{r} = \frac{50^2}{1000/\pi}$$

$$a = \frac{5\pi}{2} = 7.85 \mathrm{m/s^2}$$

The magnitude of acceleration of the car is **7.85 m/s²**.

SOLUTION 2

Given: v = 50 m/s, t = 40 s,

To find: acceleration (a)

Formula: $a = r\omega^2 = v\omega$

Calculation: From formula, $a = v\omega$

$$= v\left(\frac{2\pi}{t}\right)$$

$$= 50\left(\frac{2 \times 3.142}{40}\right)$$

$$= \frac{5}{2} \times 3.142$$

$\therefore$ a = **7.85 m/s^2**

Exercises | Q 3. (x) | Page 46

Solve the following problem.

A particle moves in a circle with constant speed of 15 m/s. The radius of the circle is 2 m. Determine the centripetal acceleration of the particle.

SOLUTION

Given: v = 15 m/s, r = 2m

To find: Centripetal acceleration (a)

Formulae: $a = \frac{v^2}{r}$

Calculation: From formula,

$$a = \frac{(15)^2}{2} = \frac{225}{2}$$

$\therefore$ a = 112.5 m/s^2

The centripetal acceleration of the particle is **112.5 m/s^2**.

Exercises | Q 3. (xi) | Page 46

Solve the following problem.

A projectile is thrown at an angle of 30° to the horizontal. What should be the range of initial velocity (u) so that its range will be between 40m and 50 m? Assume g = 10 m s^{-2}.

SOLUTION

Given: $40 \le R \le 50$, $\theta = 30°$, g = 10 m/s^2

To find: Range of initial velocity (u)

Formula: $R = \frac{u^2 \sin(2\theta)}{g}$

Calculation: From formula,

The range of initial velocity,

$$40 \le \frac{u^2 \sin(2\theta)}{g} \le 50$$

$$\therefore \frac{40g}{\sin(2\theta)} \le u^2 \le \frac{50g}{\sin(2\theta)}$$

$$\therefore \sqrt{\frac{40g}{\sin(2\theta)}} \le u \le \sqrt{\frac{50g}{\sin(2\theta)}}$$

$$\therefore \sqrt{\frac{40 \times 10}{\sin(60)}} \le u \le \sqrt{\frac{50 \times 10}{\sin(60)}}$$

$\therefore$ **21.49 m/s ≤ u ≤ 24.03 m/s**

The range of initial velocity should be between **21.49 m/s ≤ u ≤ 24.03 m/s**.